Continuous Data (1 of 2)

Aug 8, 2023.

Exploring Univariate Continuous Data

THIS CHAPTER explores how to summarize and visualize *univariate, continuous* data.

1. **Univariate** continuous data refers to data coming from **one feature or variable**, which could take on an infinite number of possible values, typically within an interval [1].

2. For instance, in the `mtcars` dataset in R, variables like `mpg` (miles per gallon), `wt` (weight), and `hp` (horsepower) epitomize continuous data. They are not limited to specific, separate numbers and can encompass any value, including decimal points, within their respective ranges. [2]

3. We will leverage the capabilities of R programming and the `dplyr` package to compute descriptive statistics in order to succinctly represent our data. Further on, the spotlight will be on visualization. With the help of the robust `ggplot` package, we will create {bee swarm plots, stem-and-leaf plots, histograms, density plots, box plots, violin plots}. These plots will not only represent our univariate continuous data but also facilitate our understanding of data distribution, outliers, and central tendency.

4. **Data**: Suppose we run the following code to prepare the `mtcars` data for subsequent analysis and save it in a tibble called `tb`.

```r
# Load the required libraries, suppressing annoying startup messages
library(tibble)
suppressPackageStartupMessages(library(dplyr))
# Read the mtcars dataset into a tibble called tb
data(mtcars)
tb <- as_tibble(mtcars)
# Convert relevant columns into factor variables
tb$cyl <- as.factor(tb$cyl) # cyl = {4,6,8}, number of cylinders
tb$am <- as.factor(tb$am) # am = {0,1}, 0:automatic, 1: manual transmission
tb$vs <- as.factor(tb$vs) # vs = {0,1}, v-shaped engine, 0:no, 1:yes
tb$gear <- as.factor(tb$gear) # gear = {3,4,5}, number of gears
```

```r
# Directly access the data columns of tb, without tb$mpg
attach(tb)
```

Measures of Central Tendency

1. In our journey of understanding data, we often turn to certain statistical tools, among which, the measures of central tendency play a pivotal role. These measures provide a way to summarize our data with a single value that represents the "center" or the "average" of our data distribution. [1]

2. Primarily, there are three measures of central tendency that we often rely on: the mean, median, and mode. [2]

3. As an illustration, here is R code to determine the `mean` and `median` of the `wt` (weight) for all vehicles:

```r
# Mean of wt
mean(tb$wt)
```

```
[1] 3.21725
```

```r
# Median of wt
median(tb$wt)
```

```
[1] 3.325
```

5. For finding the mode of the `mpg` (miles per gallon) column, we use the `mfv()` function in the `modeest` package.

```r
# Calculate mode of mpg
library(modeest)
mfv(tb$mpg) # Mode
```

```
[1] 10.4 15.2 19.2 21.0 21.4 22.8 30.4
```

6. The `mfv()` function estimates the mode using a kernel density estimator, which may not always coincide with a specific value in the dataset [4].

Measures of Variability

1. In our exploration of continuous data, we also consider measures of variability. These statistical measures provide insight into the spread or dispersion of our data points. To further illustrate the concepts we've discussed, we'll apply these measures of variability to the `mpg` column from the `mtcars` dataset.

2. **Range**: This is the difference between the highest and the lowest value in our data set. However, while `range` is easy to calculate and understand, it is sensitive to outliers, so we must interpret it carefully. The `range()` function in R provides the minimum and maximum `mpg`.

```
# Range of mpg
range(tb$mpg)
```

```
[1] 10.4 33.9
```

3. **Min and Max**: We can off course measure the minimum and maximum values, using the following simple code.

```
# Minimum mpg
min(tb$mpg)
```

```
[1] 10.4
```

```
# Maximum mpg
max(tb$mpg)
```

```
[1] 33.9
```

4. **Variance**: It is calculated as the average of the squared deviations from the mean. Larger variances suggest that the data points are more spread out around the mean. One limitation of the variance is that its units are the square of the original data's units, which can make interpretation difficult. We use the var() function to compute the variance.

```
# Variance of mpg
var(tb$mpg)
```

```
[1] 36.3241
```

5. **Standard Deviation:** This is simply the square root of the variance. Because it is in the same units as the original data, it is often easier to interpret than the variance. A larger standard deviation indicates a greater spread of data around the mean.

```
# Standard Deviation of mpg
sd(tb$mpg)
```

```
[1] 6.026948
```

6. **Interquartile Range (IQR)**: It is another measure of dispersion, especially useful when we have skewed data or outliers. It represents the range within which the central 50% of our data falls. This measure is less sensitive to extreme values than the range, variance, or standard deviation. To find the interquartile range (IQR), which provides the spread of the middle 50% of the `mpg` values, we use the `IQR()` function.

```
# Inter-Quartile Range of mpg
IQR(tb$mpg)
```

```
[1] 7.375
```

7. **Skewness** and **Kurtosis**:

- **Skewness** is a measure of the asymmetry of our data. Positive skewness indicates a distribution with a long right tail, while negative skewness indicates a distribution with a long left tail.
- **Kurtosis**, on the other hand, measures the "tailedness" of the distribution. A distribution with high kurtosis exhibits a distinct peak and heavy tails, while low kurtosis corresponds to a flatter shape.
- These two measures can be computed using the `skewness()` and `kurtosis()` functions from the `moments` package.

```
# Load moments package
suppressPackageStartupMessages(library(moments))

# Skewness of 'wt' in the mtcars dataframe
skewness(tb$wt)
```

```
[1] 0.4437855
```

```r
# Kurtosis of 'wt' in the mtcars dataframe
kurtosis(tb$wt)
```

```
[1] 3.172471
```

8. Overall, these measures of variability help us quantify the dispersion and shape of our data, offering a more complete picture when combined with measures of central tendency. [3]

Summarizing Univariate Continuous Data

1. Our primary objective in summarizing data is to gain an initial overview or snapshot of the data set we're dealing with. This fundamental analysis provides us a sense of the data's central tendency, spread, and distribution shape, which in turn guides our decision-making process for subsequent stages of data analysis.
2. In R, the `summary()` function offers a succinct summary of the selected data object. When applied to a numeric vector such as `mpg` from the `mtcars` dataset, it yields the minimum and maximum values, the first quartile (25th percentile), the median (50th percentile), the third quartile (75th percentile), and the mean.

```r
# A summary of 'mpg'
summary(tb$mpg)
```

```
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max. 
  10.40   15.43   19.20   20.09   22.80   33.90 
```

3. The `describe()` function, part of the `psych` package, goes a step further by providing a more comprehensive summary of the data. It includes additional statistics like the number of valid (non-missing) observations, the standard deviation, and metrics of skewness and kurtosis [5].

```r
suppressPackageStartupMessages(library(psych))

# A summary of 'mpg' using describe()
describe(tb$mpg)
```

```
   vars  n  mean   sd median trimmed  mad  min  max range skew kurtosis   se
X1    1 32 20.09 6.03   19.2    19.7 5.41 10.4 33.9  23.5 0.61    -0.37 1.07
```

4. Specific columns from `describe(tb$mpg)`

```
# Select specific columns from describe(mpg)
columns = c("n","mean","sd","median","min","max","skew","kurtosis")
describe(tb$mpg)[, columns]
```

```
   n  mean   sd median  min  max skew kurtosis
X1 32 20.09 6.03   19.2 10.4 33.9 0.61    -0.37
```

Summarizing an entire dataframe or tibble

1. The function `summary()` in R can also be employed to summarize the entirety of a dataframe or tibble in a comprehensive manner. [3].

```
# A summary of the tibble tb
summary(tb)
```

```
      mpg         cyl        disp             hp             drat      
 Min.   :10.40   4:11   Min.   : 71.1   Min.   : 52.0   Min.   :2.760  
 1st Qu.:15.43   6: 7   1st Qu.:120.8   1st Qu.: 96.5   1st Qu.:3.080  
 Median :19.20   8:14   Median :196.3   Median :123.0   Median :3.695  
 Mean   :20.09          Mean   :230.7   Mean   :146.7   Mean   :3.597  
 3rd Qu.:22.80          3rd Qu.:326.0   3rd Qu.:180.0   3rd Qu.:3.920  
 Max.   :33.90          Max.   :472.0   Max.   :335.0   Max.   :4.930  
       wt             qsec        vs      am     gear        carb      
 Min.   :1.513   Min.   :14.50   0:18   0:19   3:15   Min.   :1.000  
 1st Qu.:2.581   1st Qu.:16.89   1:14   1:13   4:12   1st Qu.:2.000  
 Median :3.325   Median :17.71                 5: 5   Median :2.000  
 Mean   :3.217   Mean   :17.85                        Mean   :2.812  
 3rd Qu.:3.610   3rd Qu.:18.90                        3rd Qu.:4.000  
 Max.   :5.424   Max.   :22.90                        Max.   :8.000  
```

2. **Discussion**

- For numeric columns, `summary()` delivers a six-number summary that includes minimum, first quartile (Q1 or 25th percentile), median (Q2 or 50th percentile), mean, third quartile (Q3 or 75th percentile), and maximum. This gives a broad understanding of the central tendency and dispersion of the data within each numeric column.

- For categorical (factor) columns, `summary()` generates the counts of each category level. The output of this code is essentially a comprehensive snapshot of the `tb` tibble, enabling us to quickly understand the nature of our data. [3]

3. To obtain a more detailed statistical summary of an entire dataframe or tibble, we can employ the `describe()` function from the `psych` package [2].

```
# Select specific columns from describe(mpg)
columns = c("n","mean","sd","median","min","max","skew","kurtosis")
describe(tb)[, columns]
```

```
       n   mean     sd median   min    max  skew kurtosis
mpg   32  20.09   6.03  19.20 10.40  33.90  0.61    -0.37
cyl*  32   2.09   0.89   2.00  1.00   3.00 -0.17    -1.76
disp  32 230.72 123.94 196.30 71.10 472.00  0.38    -1.21
hp    32 146.69  68.56 123.00 52.00 335.00  0.73    -0.14
drat  32   3.60   0.53   3.70  2.76   4.93  0.27    -0.71
wt    32   3.22   0.98   3.33  1.51   5.42  0.42    -0.02
qsec  32  17.85   1.79  17.71 14.50  22.90  0.37     0.34
vs*   32   1.44   0.50   1.00  1.00   2.00  0.24    -2.00
am*   32   1.41   0.50   1.00  1.00   2.00  0.36    -1.92
gear* 32   1.69   0.74   2.00  1.00   3.00  0.53    -1.07
carb  32   2.81   1.62   2.00  1.00   8.00  1.05     1.26
```

4. **Discussion:**

- The `describe()` function analyzes each column in the provided tibble individually and outputs a range of useful statistics. For numeric columns, it offers count, mean, standard deviation, trimmed mean, minimum and maximum values, range, skewness, and kurtosis among others.

- For non-numeric or factor columns, the `describe()` function still provides a count of elements but defaults to `NA` for the rest of the statistics, as these metrics are not applicable.

Visualizing Univariate Continuous Data

- In our journey to explore and understand univariate continuous data, visualizations act as our valuable companions. Visual graphics provide us with an instant and clear understanding of the underlying data patterns and distributions that may otherwise be challenging to discern from raw numerical data.

- Let's take a closer look at some of the most effective ways of visualizing univariate continuous data, including

(i) Bee Swarm plots;

(ii) Stem-and-Leaf plots

(iii) Histograms;

(iv) PDF and CDF Density plots;

(v) Box plots;

(vi) Violin plots;

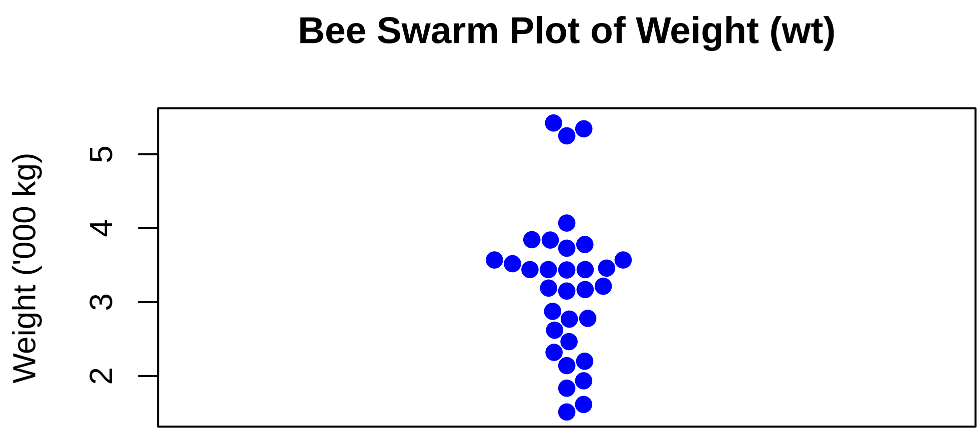
(vii) Quantile-Quantile (Q-Q) Plots

Bee Swarm plot

1. A Bee Swarm plot is a one-dimensional scatter plot that reduces overlap and provides a better representation of the distribution of individual data points (Ellis, 2011). This type of plot provides a more detailed view of the data, particularly for smaller data sets.
2. It displays all of the individual data points along with a visual representation of their distribution. It can be useful for displaying the distribution of small datasets.

```
# Load the beeswarm package
library(beeswarm)

# Create a bee swarm plot of wt column
beeswarm(tb$wt,
         main="Bee Swarm Plot of Weight (wt)",
         ylab = "Weight ('000 kg)",
         pch=16,
         cex=1.2,
         col="blue")
```



- In the above code, we load the `beeswarm` package using the `library()` function.
- We then create a bee swarm plot of the `wt` column using the `beeswarm()` function.
- The `main` argument is used to specify the title of the plot.
- The `pch` argument is used to set the type of points to be plotted, and the `cex` argument is used to set the size of the points.
- The `col` argument is used to set the color of the points.
- The resulting plot will display the individual `wt` values in the dataset as points on a horizontal axis, with no overlap between points. This provides a visual representation of the distribution of the data, as well as any outliers or gaps in the data.

Stem-and-Leaf Plots

1. Stem-and-leaf plots serve as an efficient tool for visualizing the distribution of data, particularly when working with small to medium-sized datasets. The method involves breaking down each data point into a "stem" and a "leaf", with the "stem" representing the primary digit(s) and the "leaf" embodying the subsequent digit(s) [7]
2. We can utilize the `stem()` function in R to devise stem-and-leaf plots. Here's how we can apply it to the `mpg` column in our `tb` tibble:

```
stem(tb$mpg)
```

```
The decimal point is at the |

10 | 44
12 | 3
```

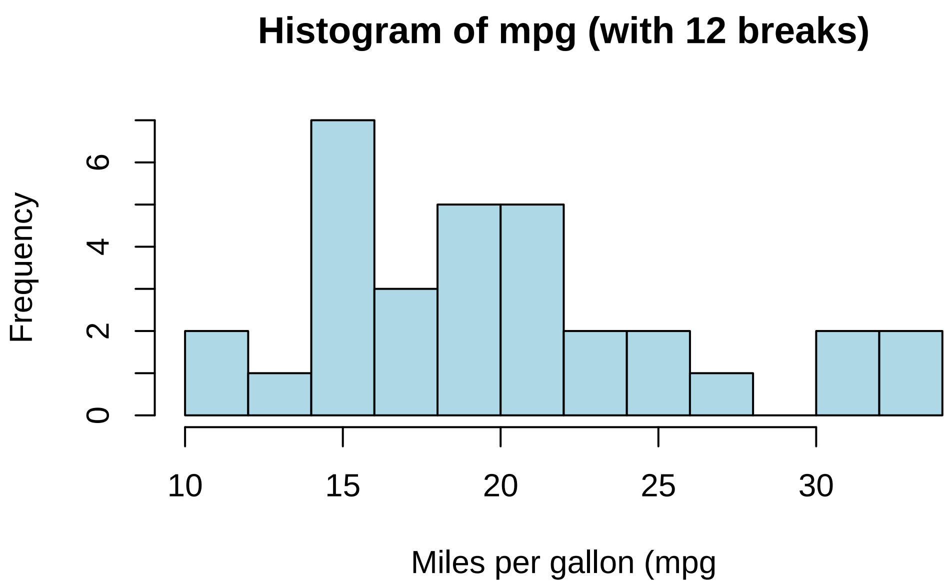
```
14 | 3702258
16 | 438
18 | 17227
20 | 00445
22 | 88
24 | 4
26 | 03
28 |
30 | 44
32 | 49
```

3. In the resulting plot, the vertical bar ("|") symbolizes the decimal point's location.
4. This visual representation enables us to swiftly assess the data's distribution, the center, and the spread, in a fashion similar to a histogram. However, unlike a histogram, a stem-and-leaf plot retains the original data to a certain degree, providing more granular detail.

Histogram

1. A histogram is a graphical representation showcasing the *frequency* of discrete or grouped data points within a dataset.
2. It splits the data into equal-width bins, with the height of each bar matching the frequency of data points in each respective bin.
3. It serves as a valuable tool for demonstrating the distribution shape of the data. In R, we can construct a histogram using the `hist()` function and control its appearance. The final histogram visually depicts the frequency of `mpg` values in the dataset, where each bar represents the count of observations within a specific range of values. [7]

```r
# Create a histogram of mpg column with a specific number of bins of equal width
hist(tb$mpg,
     breaks = 12, # This creates 12 bins of equal width
     main="Histogram of mpg (with 12 breaks)",
     xlab="Miles per gallon (mpg",
     col="lightblue",
     border="black")
```

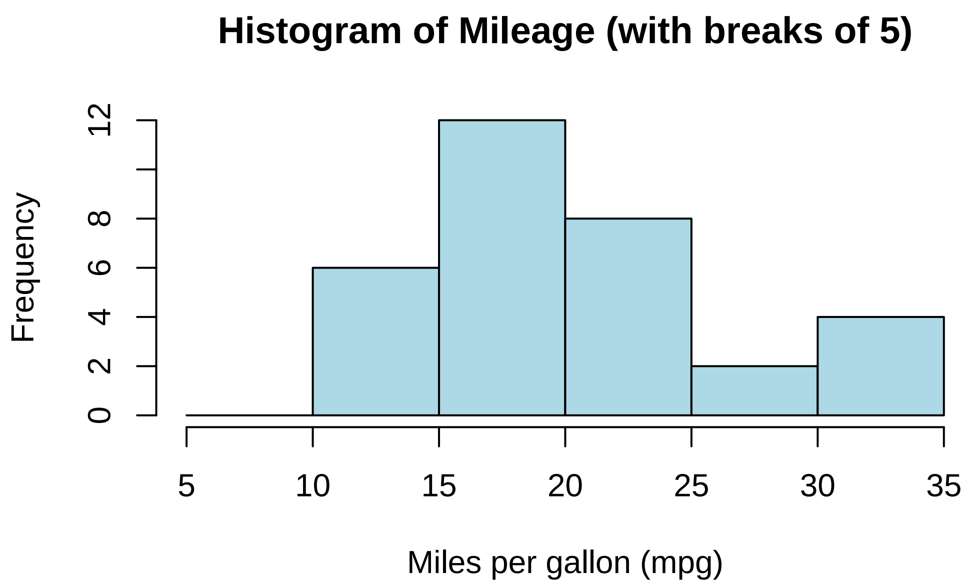


4. **Discussion:**

- This code generates a histogram of `mpg` using the `hist()` function. The `main` argument denotes the plot's title, while the `xlab` argument labels the x-axis.
- We use the `col` argument to specify the color of the histogram bars, and the `border` argument to determine the color of the bar borders.
- We can control the *number of bins* or the ranges of the bins in a histogram using the `breaks` argument inside the `hist()` function:

5. We can alternately specify the *ranges of the bins*:

```
# Create a histogram of mpg column with specific bin ranges
hist(tb$mpg,
     breaks = seq(5, 35, by = 5), # This creates bins with ranges 10-15, 15-20, etc.
     main="Histogram of Mileage (with breaks of 5)",
     xlab="Miles per gallon (mpg)",
     col="lightblue",
     border="black")
```



6. **Discussion:**

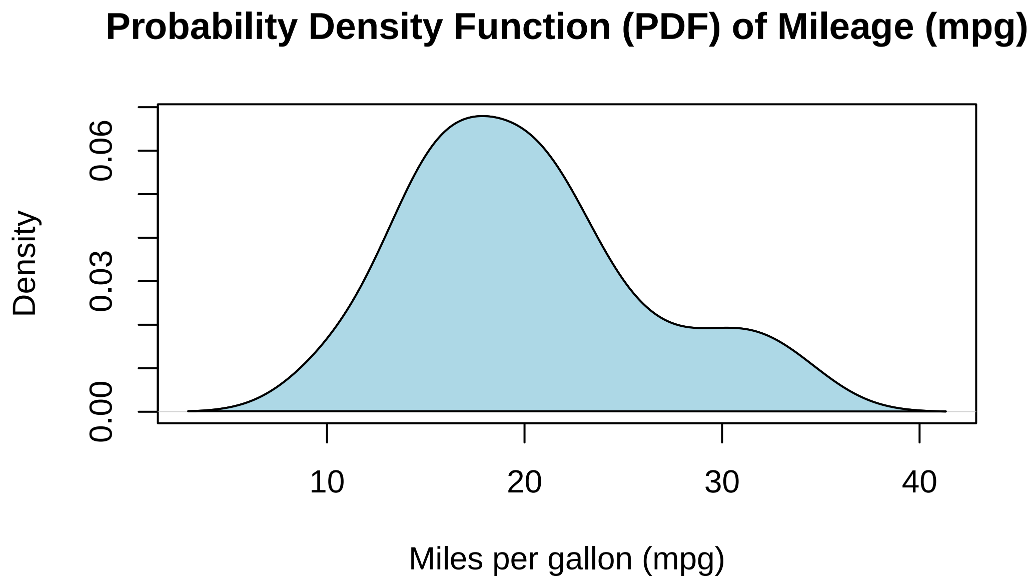
- The breaks argument uses the `seq()` function to create a sequence of break points from 5 to 35, with a step of 5.
- This results in bins with ranges 5-10, 10-15, 15-20, 20-25, 25-30, and 30-35. [7]

Probability Density Function (PDF) plot

1. Smoothed approximations of histograms are often represented by density plots, as they assist in offering an estimation of the underlying continuous probability distribution of a given dataset. [7]
2. Compared to histograms, these plots often present superior accuracy and aesthetic appeal, and they eliminate the need for arbitrary bin selection. A density plot shares several similarities with a histogram. However, instead of presenting the frequency of individual values, it conveys the probability density of the dataset. [7]

```
# Calculate density
dens <- density(mtcars$mpg)
# Create a density plot
plot(dens,
     main = "Probability Density Function (PDF) of Mileage (mpg)",
     xlab = "Miles per gallon (mpg)",
)
# Add a polygon to fill under the density curve
```

```
polygon(dens, col = "lightblue", border = "black")
```



3. **Discussion:**

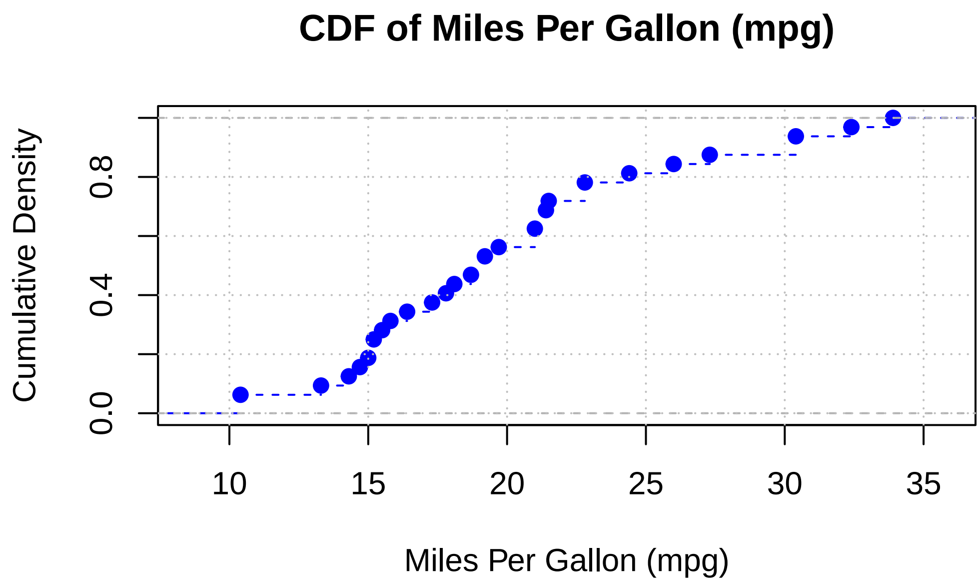
- We use the `density()` function to generate a PDF plot for the `mpg` column.
- Here, we utilize the `plot()` function to graph the resulting density object.
- The `main` argument stipulates the title of the plot, while the `xlab` argument designates the label for the x-axis.
- Through the `polygon()` function, we determine the shaded color.
- The final plot displays the probability density of `mpg` values, using the curve to signify the data distribution. [7]

Cumulative Distribution Function (CDF) Plot

1. CDF plots visualize the fraction of data points that are less than or equal to a specified value on the x-axis [3].
2. They facilitate easy representation of the median, percentiles, and spread.
3. In R, we can employ the `ecdf()` function to generate a CDF plot.

```
# Create a CDF plot of mpg column
plot(ecdf(tb$mpg),
```

```
     main = "CDF of Miles Per Gallon (mpg)",
     xlab = "Miles Per Gallon (mpg)",
     ylab = "Cumulative Density",
     col = "blue",       # Line color
     lty = 2,
     )
grid(col = "gray", lty = "dotted")  # Add a grid to the plot
```



4. **Discussion:**

- `ecdf(tb$mpg)`: The function `ecdf()` computes the empirical cumulative distribution function (CDF) for the mpg column from the `tb` data frame.
- `plot(ecdf(tb$mpg))`: Plots the CDF of the mpg column.
- `main, xlab, ylab`: Set the title and axis labels.
- `col = "blue"`: Colors the plot line blue; `lty = 2`: Uses a dashed line.
- `grid(col = "gray", lty = "dotted")`: Adds a gray, dotted grid to the plot.

5. Computing and Inversing CDF Values

- The following code demonstrates how to determine the cumulative distribution function (CDF) value for a given `mpg` using `ecdf()` and how to find the `mpg` value corresponding to a specific CDF with `quantile()`.
- Suppose we want to identify the CDF at `mpg = 20`, here is how we do it:

```
# Generate the empirical cumulative distribution function
ecdf_func <- ecdf(tb$mpg)

# Derive the CDF for mpg = 20
ecdf_func(20)
```

```
[1] 0.5625
```

- If we're interested in knowing the `mpg` value that corresponds to a certain CDF value, the `quantile()` function comes to our aid. For instance, we can obtain the `mpg` value associated with a CDF of `0.6` as follows:

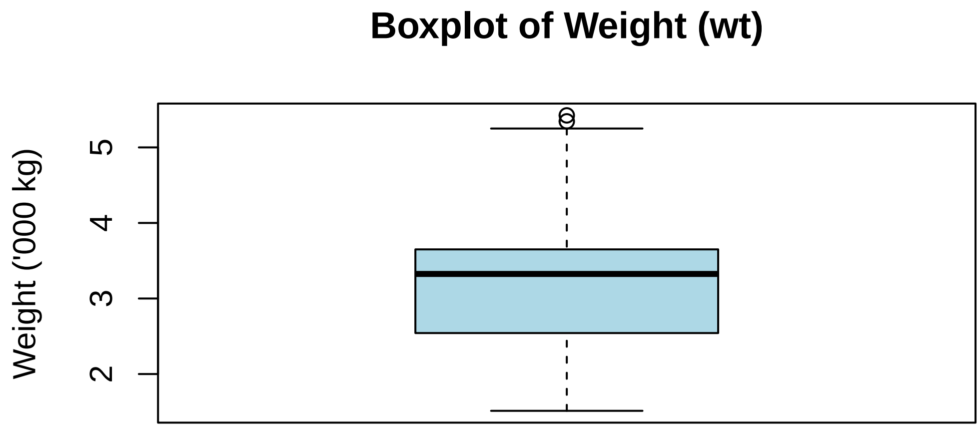
```
# Discover the mpg corresponding to CDF = 0.6
quantile(tb$mpg, 0.6)
```

```
60%
 21
```

Boxplot

1. Box-and-whisker plots, commonly known as box plots, are crucial graphical instruments for illustrating a distribution's center, spread, and potential outliers [5].
2. Here is sample code to generate a boxplot of `wt` (Weight) of the cars .

```
boxplot(tb$wt,
        xlab = "",
        ylab = "Weight ('000 kg)",
        main = "Boxplot of Weight (wt)",
        col = "lightblue"
       )
```



3. The box plot's construction involves the use of an interquartile range (IQR) represented by a box, which contains the middle 50% of the dataset.
4. The box's internal line signifies the median, while the "whiskers" reach out to the smallest and largest observations within a distance of 1.5 times the IQR.
5. The whiskers extend to the minimum and maximum non-outlier values, or 1.5 times the interquartile range beyond the quartiles, whichever is shorter.
6. Any points outside of the whiskers are considered outliers and are plotted individually.

Violin plot

1. Violin plots are a compelling tool to merge the benefits of box plots and kernel density plots and enable us to depict a detailed view of data distribution.
2. These plots exhibit the probability density at different values, where the plot's breadth represents the density or frequency of data points. More extensive areas denote a higher aggregation of data points Akin to a box plot, a violin plot provides a visual display of the entire data distribution via a kernel density estimate, as opposed to just presenting the quartiles [5].
3. The `vioplot()` function, part of the `vioplot` package in R, allows us to create such a violin plot.

```r
library(vioplot)
```

```
Loading required package: sm
```

```
Package 'sm', version 2.2-5.7: type help(sm) for summary information
```

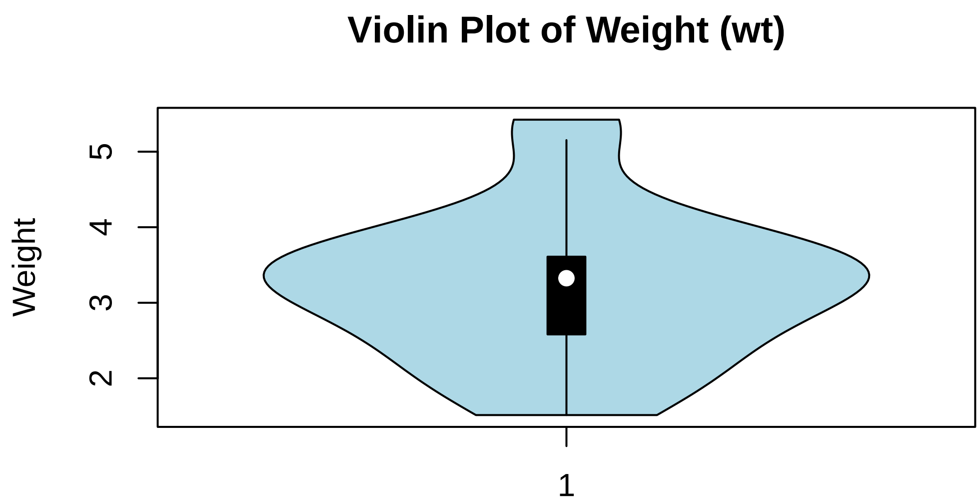
```
Loading required package: zoo


Attaching package: 'zoo'


The following objects are masked from 'package:base':

    as.Date, as.Date.numeric
```

```r
# Constructing a violin plot for the wt
vioplot(tb$wt,
        main="Violin Plot of Weight (wt)",
        ylab="Weight",
        col = "lightblue")
```



- In this code, the `vioplot()` function crafts a violin plot for the mpg variable. We use the `main` argument to assign the plot's title and the `ylab` argument to designate the label for the y-axis.
- The resulting plot unveils the entire `wt` data distribution, with a kernel density estimate indicating the concentration of data points at different sections.
- Lastly, the plot incorporates the median, quartiles, and any outliers present in the data.

Quantile-Quantile (Q-Q) Plots

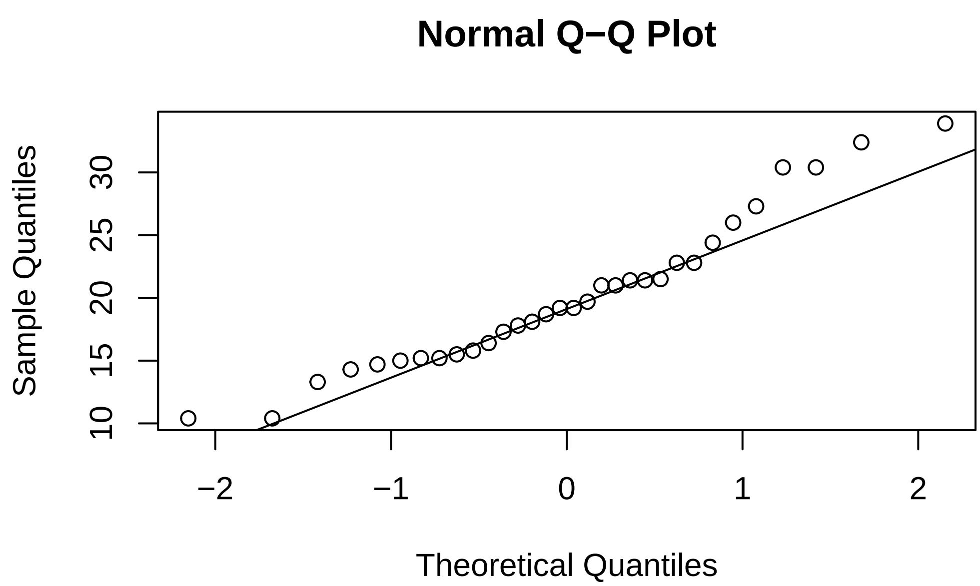
1. Quantile-Quantile plots, commonly referred to as Q-Q plots, are a visual tool we use to check if data follows a particular distribution, like a normal distribution.

2. Suppose we order a data column from the smallest to the biggest value, and each data point gets a *score* based on its position. This is what we call a *quantile*. Now, imagine a perfectly normal distribution doing the same thing. In a Q-Q plot, we compare our data's scores to the scores from the ideal normal distribution.

3. If our data aligns with the normal distribution, the points in the Q-Q plot will form a straight line. But if our data doesn't follow the normal distribution, the points will stray from the line. This way, the Q-Q plot gives us an intuitive, visual way to decide if our data is normally distributed or not [7].

4. In R, we can use the `qqnorm()` function to create the plot and the `qqline()` function to add the reference line. If the points lie close to the reference line, it's a good indication that our data is normally distributed.

```
# Generate a Q-Q plot for 'mpg' column
qqnorm(tb$mpg)
# Add a reference line to the plot
qqline(tb$mpg)
```



5. This approach isn't limited to normal distributions. We can compare our data with other distributions too, which makes Q-Q plots a versatile tool for understanding our data's behavior.

Summary of Chapter 12 – Continuous Data (1 of 2)

This section of the book examines continuous univariate data, focusing on single variables in the 'mtcars' dataset using R's dplyr and ggplot packages. We employ R's inherent functions and the 'modeest' package to compute the mean, median, and mode, alongside variability measures like range, variance, and standard deviation.

We use R's 'summary()' and the psych package's 'describe()' functions to create succinct and detailed overviews of our data, providing insights into its central tendency, spread, and distribution shape. These functions can also summarise an entire dataframe or tibble, setting the stage for future analysis.

Visualisations are key to understanding data patterns and distributions. We use bee swarm plots, box plots, violin plots, histograms, and density plots. Bee swarm plots, using the `beeswarm()` function, show all data points and their distributions. Stem-and-leaf plots, created using the `stem()` function, provide a quick evaluation of the data's distribution.

Histograms, constructed with the `hist()` function, and density plots, using the `density()` function, display data frequency and smoothed approximations respectively. Cumulative Distribution Function (CDF) plots, via the `ecdf()` function, show the proportion of data points equal to or less than specific values.

Box plots, made with the `boxplot` function, highlight the distribution's center, spread, and outliers. Violin plots, via the `vioplot()` function, merge box plots and kernel density plots to display data density. Lastly, Q-Q plots, created using `qqnorm()` and `qqline()`, verify if data follows a normal distribution.

In sum, this chapter presents key R functions and techniques for visualising continuous univariate data, providing valuable insights into data patterns and distributions.

References

[1]

Moore, D. S., McCabe, G. P., & Craig, B. A. (2012). Introduction to the Practice of Statistics. Freeman.

Triola, M. (2017). Elementary Statistics. Pearson.

Gravetter, F. J., & Wallnau, L. B. (2016). Statistics for the Behavioral Sciences. Cengage Learning.

[2]

Downey, A. B. (2014). Think Stats: Exploratory Data Analysis. O'Reilly Media.

[3]

Field, A., Miles, J., & Field, Z. (2012). Discovering statistics using R. Sage Publications.

R Core Team (2020). R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. URL https://www.R-project.org/.

[4]

Bogaert, P. (2021). "A Comparison of Kernel Density Estimators." Computational Statistics & Data Analysis, 77, 402-413.

[5]

Revelle, W. (2020). psych: Procedures for Psychological, Psychometric, and Personality Research. Northwestern University, Evanston, Illinois. R package version 2.0.12, https://CRAN.R-project.org/package=psych.

Tukey, J. W. (1977). Exploratory data analysis. Addison-Wesley.

[6]

Ellis, K. (2011). Beeswarm: The Bee Swarm Plot, an Alternative to Stripchart. R package version 0.2.3.

Hyndman, R. J., & Fan, Y. (1996). Sample quantiles in statistical packages. The American Statistician, 50(4), 361-365.

[7] Scott, D. W. (1979). On optimal and data-based histograms. Biometrika, 66(3), 605-610.

Wand, M. P., & Jones, M. C. (1995). Kernel Smoothing. Chapman and Hall/CRC.

[8]

McGill, R., Tukey, J. W., & Larsen, W. A. (1978). Variations of Box Plots. The American Statistician, 32(1), 12-16.

[9]

Hintze, J. L., & Nelson, R. D. (1998). Violin Plots: A Box Plot-Density Trace Synergism. The American Statistician, 52(2), 181-184.

[10]

Thode Jr, H. C. (2002). Testing for normality. CRC press.